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A UNIFIED APPROACH FOR REAL TIME DYNAMIC HYBRID SIMULATION

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ABSTRACT

This paper proposes a unified formulation for Real Time Dynamic Hybrid Simulation (RTDHS). By choosing defining different values of splitting coefficient matrices, one may describe and execute various modern seismic testing methods, such as real time pseudo-dynamic testing, effective force testing and shake table testing. This paper reviews first the concept of the Real Time Dynamic Hybrid Simulation and then presents the unified formulation with a detailed discussion of the splitting coefficient matrices. Hardware components necessary to implement the unified formulation RTDHS are integrated into a unified test platform, which includes Structural and Seismic Testing Controllers; Data Acquisition and Information Streaming and Real Time Hybrid Simulation Controllers. While a number of tests were performed in medium scale, a small-scale pilot setup including a one-story shear model, an actuator and a one directional shake table were constructed for the proof-of-concept tests of the proposed unified approach. Test results, which validate the concept of the proposed unified formulation and the feasibility of the corresponding operating test platform for RTDHS are discussed.

Introduction

Real time seismic testing of very large full-scale specimens and models is currently possible in a few engineering laboratories. However, full-scale laboratory testing of entire civil engineering structures (e.g., cable-stay bridges, multi-story buildings, industrial facilities, and pipeline distribution systems) is not likely to occur in the near future due to the prohibitive costs that would be associated with such testing. It is believed that the best approach to generate experimentally the data needed for the development of reliable and accurate computational models is to complement large-scale substructure model testing with real-time interactive computational simulations of entire systems. The intent of Real Time Dynamic Hybrid Simulation (RTDHS) is to test large structures or substructures using shake tables, while simultaneously applying actively controlled dynamic forces at the boundary of the specimens – forces that simulate the behavior and interactions of the remainder of the structure (Reinhorn et al. 2003). Conceptually, RTDHS allows a researcher to focus on specific problems represented in the substructure under the most realistic conditions using emerging computational power in tandem with state-of-the-art control systems. Such procedures and configurations may extend significantly the testing capabilities by integrating large-size physical components into virtual complete systems of theoretically unlimited size and configuration.

The structure to be simulated in a RTDHS is divided into a physical experimental substructure and one or

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more computational substructures, represented in red and black respectively in Fig. 1. The interface forces between the physical and computational substructures are imposed by actuators and resulting displacements and velocities are fed back to the computational engine. The earthquake ground motion, or motion of other computational substructures, is applied to the experimental substructure by shake tables. Fig. 1 shows a schematic of the RTDHS testing system. The right side of the plot shows the computational hardware infrastructure required for the implementation of the forces and motions at the interface of the physical experimental and computational substructures.

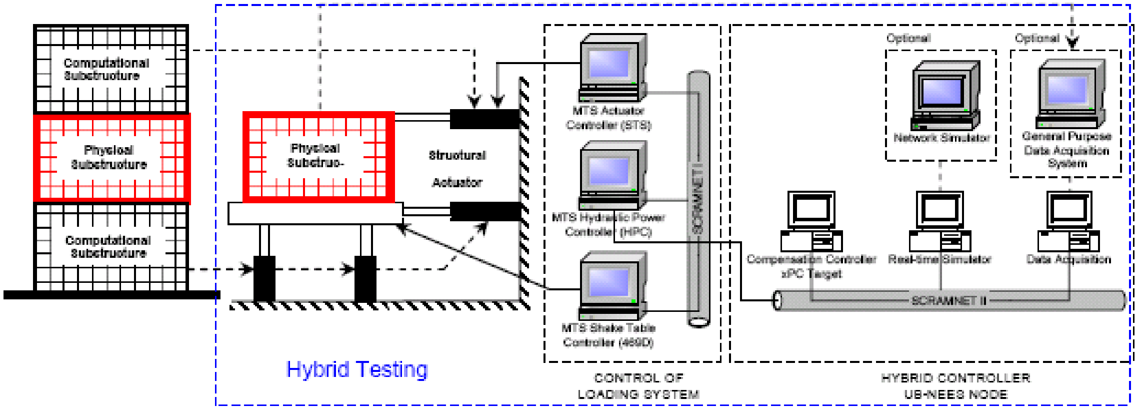


Figure 1. Schematic of Real Time Dynamic Hybrid Simulation (RTDHS) testing.

While the development of RTDHS progresses, the simulation techniques proposed can be extended to create a unified technique comprised of several seismic simulation methods such as dynamic simulation, pseudo-dynamic simulation, or even a hybrid combination of dynamic and pseudo-dynamic simulations. All these various simulation methods can be implemented by a selection of splitting coefficient matrices in the unified formulation as discussed in the next section.

Unified Formulation

A derivation for substructure formulation in the Real Time Dynamic Hybrid Simulation can be obtained by partitioning the dynamic property matrices (mass, damping and stiffness matrices) of the equation of motion describing the whole structural model, the equation of motion for the experimental substructure as (Shao, 2006):

$$\mathbf{M}_{ep}\ddot{\mathbf{X}}_{ep} + \mathbf{C}_{ep}\dot{\mathbf{X}}_{ep} + \mathbf{K}_{ep}\mathbf{X}_{ep} = -\mathbf{M}_{ep}\mathbf{R}_{e}\ddot{\mathbf{u}}_{g} + \mathbf{T}_{ep} \tag{1}$$

where $\mathbf{M}_{ep}$, $\mathbf{C}_{ep}$ and $\mathbf{K}_{ep}$ are the mass, damping and stiffness matrices of the experimental substructure respectively, and $\ddot{\mathbf{X}}_{ep}$, $\dot{\mathbf{X}}_{ep}$ and $\mathbf{X}_{ep}$ are vectors of experimental substructure's accelerations, velocities and displacements/rotations associated with each degree-of-freedom relative to the ground reference frame. The input to the experimental substructure described by the right side of Eq. 1 contains two parts. One is the base acceleration input $\ddot{\mathbf{U}}_g$ and the other the interface force vector $\boldsymbol{T}_{ep}$, resulting from the interaction between the computational and physical experimental substructures. When the RTDHS was first proposed, the base acceleration input was designated to be applied by the shake tables and the interface forces to be applied by the force controlled dynamic actuators. However, there are several other ways to realize the dynamic loads required to be applied to the experimental substructure in order to simulate the boundary effects. These alternatives can be generalized in one unified formulation and discussed in two steps as follows.

Hybrid Simulation of Dynamic and Pseudo-Dynamic Testing

In a conventional dynamic testing as described by Eq. 1, the full mass $\mathbf{M}_{ep}$ of the experimental substructure is included in the physical specimen. The inertia forces are developed naturally in the physical specimen during the testing. However, in structures that are large in respect to the experimental equipment, such masses may be difficult to build and support. To overcome these limitations, a portion of the mass can be modeled numerically in a computer in order to reduce the size of the physical mass to be fabricated, installed, and tested (see also Kausel, 1998 and Chen *et al*. 2006). The mass that is modeled analytically is defined as *virtual mass.* A mass splitting coefficient matrix $\boldsymbol{\alpha}_m$ is defined as a diagonal matrix consisting of the ratio of the virtual mass ($\mathbf{M}^v_{ep}$) and the total mass of the experimental substructure ($\mathbf{M}_{ep}$) required in the simulation.

$$\boldsymbol{\alpha}_m = \mathbf{M}^v{}_{ep} \cdot \mathbf{M}^{-1}{}_{ep} = \left(\mathbf{M}_{ep} - \mathbf{M}^p{}_{ep}\right) \cdot \mathbf{M}^{-1}{}_{ep} \tag{2}$$

The *physical mass* matrix $\mathbf{M}^p{}_{ep}$ can be expressed therefore as:

$$\mathbf{M}^p{}_{ep} = \left(\mathbf{E} - \boldsymbol{\alpha}_m\right)\mathbf{M}_{ep} \tag{3}$$

in which **E** is a diagonal identity matrix. By introducing the mass splitting coefficient matrix into Eq. 1, the equation of motion for the experimental substructure with only the physical mass is derived as:

$$\mathbf{M}^p{}_{ep}\ddot{\mathbf{x}}_{ep} + \mathbf{C}_{ep}\dot{\mathbf{x}}_{ep} + \mathbf{K}_{ep}\mathbf{x}_{ep} = -\mathbf{M}^p{}_{ep}\mathbf{R}_e\ddot{\mathbf{u}}_g + \mathbf{T}''_{ep} \tag{4}$$

where $\mathbf{T}''_{ep} = -\boldsymbol{\alpha}_m\mathbf{M}_{ep}\left(\mathbf{R}_e\ddot{\mathbf{u}}_g + \ddot{\mathbf{x}}_{ep}\right) + \mathbf{T}_{ep}$ is the new force vector which must be applied to the boundaries of the reduced mass specimen during the experiment. This includes the additional inertial force related to the virtual mass of the experimental substructure. Note that the force vector includes either all, or a portion of the inertia forces, depending of the magnitude of $\boldsymbol{\alpha}_m$. Each diagonal entry of the term $\boldsymbol{\alpha}_m$ can assume a value from zero to unity. Several cases should be noted:

- When $\boldsymbol{\alpha}_m = \mathbf{0}$, the entire mass is present in the physical experimental substructure, and no virtual mass is present in the numerical model. Therefore, no supplementary inertia forces need to be applied to the specimen. This test condition, with full physical mass, is defined as a *Dynamic Test*. The total inertia force is developed physically, (or "naturally"), during the test.

- When $\boldsymbol{\alpha}_m = \mathbf{E}$, represents the case of a mass less specimen; all the inertia force is numerically simulated in the computer and should be applied to the specimen as an external force. This type of test is known as a *Pseudo-dynamic Test.* This is a static test with inertial effects realized numerically instead of physically.

- When $\mathbf{0} < \boldsymbol{\alpha}_m < \mathbf{E}$, the total mass is divided between a physical mass attached to the test specimen and a virtual mass (not all the diagonal entries in $\boldsymbol{\alpha}_m$ equal to zero or unity). This is defined as a *Quasi-dynamic Test* (a hybrid simulation of dynamic and pseudo-dynamic testing). A portion of the inertia forces are simulated numerically and applied as external forces while a portion develops naturally. This method allows for the imposition of dynamic effects when the testing facilities have limited size.

Simultaneous Loading Using Shake Tables and Actuators

For an experimental substructure with multi degrees of freedom (MDOF) as described in Eq. 1, the dynamic load can be simultaneously applied with shake tables and structural actuators. The load sharing is determined by a frequency dependent splitting coefficient matrix $\boldsymbol{\alpha}_l(s)$. The equation of motion can be rewritten, splitting the contribution of the base acceleration as follows:

$$\mathbf{M}_{ep}\ddot{\mathbf{x}}_{ep} + \mathbf{C}_{ep}\dot{\mathbf{x}}_{ep} + \mathbf{K}_{ep}\mathbf{x}_{ep} = -\mathbf{M}_{ep}\mathbf{R}_e\left(\mathbf{E} - \boldsymbol{\alpha}_l(s)\right)\ddot{\mathbf{u}}_g + \left(\mathbf{T}_{ep} - \mathbf{M}_{ep}\mathbf{R}_e\boldsymbol{\alpha}_l(s)\ddot{\mathbf{u}}_g\right) \tag{5}$$

The ground acceleration is separated into two components, with one component assigned to the base excitation device (shake table) and the other to external dynamic actuators connected to the structure.

Several cases are notable:

- When $\boldsymbol{\alpha}_l(s) = \mathbf{0}$, the shake table (or base) does not move and the entire dynamic load is applied to the experimental substructure using actuators attached to the structure at the effective interface degrees of freedom (DOFs). This is also known as the *Effective Force Testing* method. This method of testing can be performed with the whole structure, or using substructures.

- When $\boldsymbol{\alpha}_l(s) = \mathbf{E}$, the ground motion is applied at the base (shake table) without contribution of effective forces. For substructure testing, the interface forces with the complementary computational substructure can be introduced by actuators at the appropriate interface DOFs shown by term $\boldsymbol{T}_{ep}$ in Eq. 1. This is the conventional *Real Time Dynamic Hybrid Testing*.

- Besides the above two special cases, the ground acceleration (base motion), or its effects, can be applied in part by shake table (or another form of base movement) and in part by actuators. Several strategies may be used for splitting the driving function between the shake table and the dynamic actuators as proposed by Kausel (1998). In fact, the characteristics of the splitting coefficients can be chosen to optimize the total power needed by the testing system or to achieve other mechanical advantages.

Unified Formulation for Experimental Substructure Hybrid Testing

Combining the above two load splitting approaches presented in Eqs. 4 and 5, a unified formulation can be obtained to determine the load configurations applied to the experimental substructure during the hybrid testing:

$$\mathbf{M}^p{}_{ep}\ddot{\mathbf{x}}_{ep} + \mathbf{C}_{ep}\dot{\mathbf{x}}_{ep} + \mathbf{K}_{ep}\mathbf{x}_{ep} = -\mathbf{M}^p{}_{ep}\mathbf{R}_e\left(\mathbf{E} - \boldsymbol{\alpha}_l(s)\right)\ddot{\mathbf{u}}_g + \left(\mathbf{T}_{ep} - \boldsymbol{\alpha}_m\mathbf{M}_{ep}\left(\mathbf{R}_e\ddot{\mathbf{u}}_g + \ddot{\mathbf{x}}_{ep}\right) - \mathbf{M}^p{}_{ep}\mathbf{R}_e\boldsymbol{\alpha}_l(s)\ddot{\mathbf{u}}_g\right) \tag{6}$$

Table 1. Experimental Substructure in Real Time Dynamic Hybrid Simulation.

STRUCTURAL TEST MODEL		**TOTAL DYNAMIC LOAD**		
$\mathbf{M}^p{}_{ep}\ddot{\mathbf{x}}_{ep} + \mathbf{C}_{ep}\dot{\mathbf{x}}_{ep} + \mathbf{K}_{ep}\mathbf{x}_{ep}$		$-\mathbf{M}^p{}_{ep}\mathbf{R}_e\left(\mathbf{E} - \boldsymbol{\alpha}_l(s)\right)\ddot{\mathbf{u}}_g + \left(\mathbf{T}_{ep} - \boldsymbol{\alpha}_m\mathbf{M}_{ep}\left(\mathbf{R}_e\ddot{\mathbf{u}}_g + \ddot{\mathbf{x}}_{ep}\right) - \mathbf{M}^p{}_{ep}\mathbf{R}_e\boldsymbol{\alpha}_l(s)\ddot{\mathbf{u}}_g\right)$		
Test Type	***Structural Test Model***		***Table Acceleration***	***Actuators Forces***
Pseudo-Dynamic Testing $(\boldsymbol{\alpha}_m = \mathbf{E})$	$\mathbf{C}_{ep}\dot{\mathbf{x}}_{ep} + \mathbf{K}_{ep}\mathbf{x}_{ep}$	$\boldsymbol{\alpha}_l(s) = \mathbf{E}$	$\mathbf{0}$	$\mathbf{T}_{ep} - \mathbf{M}_{ep}\left(\mathbf{R}_e\ddot{\mathbf{u}}_g + \ddot{\mathbf{x}}_{ep}\right)$
Dynamic Testing $(\boldsymbol{\alpha}_m = \mathbf{0})$	$\mathbf{M}_{ep}\ddot{\mathbf{x}}_{ep} + \mathbf{C}_{ep}\dot{\mathbf{x}}_{ep} + \mathbf{K}_{ep}\mathbf{x}_{ep}$	$\boldsymbol{\alpha}_l(s) = \mathbf{E}$	$\mathbf{0}$	$\mathbf{T}_{ep} - \mathbf{M}_{ep}\mathbf{R}_e\ddot{\mathbf{u}}_g$
		$\boldsymbol{\alpha}_l(s) = \mathbf{0}$	$\ddot{\mathbf{u}}_g$	$\mathbf{T}_{ep}$
		$\mathbf{0} < \boldsymbol{\alpha}_l(s) < \mathbf{E}$	$\left(\mathbf{E} - \boldsymbol{\alpha}_l(s)\right)\ddot{\mathbf{u}}_g$	$\mathbf{T}_{ep} - \mathbf{M}^p{}_{ep}\mathbf{R}_e\boldsymbol{\alpha}_l(s)\ddot{\mathbf{u}}_g$
Quasi-Dynamic Testing $(\mathbf{0} < \boldsymbol{\alpha}_m < \mathbf{E})$	$\mathbf{M}^p{}_{ep}\ddot{\mathbf{x}}_{ep} + \mathbf{C}_{ep}\dot{\mathbf{x}}_{ep} + \mathbf{K}_{ep}\mathbf{x}_{ep}$	$\boldsymbol{\alpha}_l(s) = \mathbf{E}$	$\mathbf{0}$	$-\mathbf{M}^p{}_{ep}\mathbf{R}_e\ddot{\mathbf{u}}_g + \mathbf{T}''_{ep}$
		$\boldsymbol{\alpha}_l(s) = \mathbf{0}$	$\ddot{\mathbf{u}}_g$	$\mathbf{T}''_{ep}$
		$\mathbf{0} < \boldsymbol{\alpha}_l(s) < \mathbf{E}$	$\left(\mathbf{E} - \boldsymbol{\alpha}_l(s)\right)\ddot{\mathbf{u}}_g$	$\mathbf{T}''_{ep} - \mathbf{M}^p{}_{ep}\mathbf{R}_e\boldsymbol{\alpha}_l(s)\ddot{\mathbf{u}}_g$

Although some cases were discussed previously, a list of all possible implementation cases derived from Eq. 6 is summarized in Table 1. The three types of tests (pseudo-dynamic, dynamic and quasi-dynamic), and the associated load application split between the shake tables and the dynamic actuators, are identified. For the pseudo-dynamic testing in row 4 of Table 1, the physical specimen is usually without mass, and the whole set of inertial effects is applied by the actuators, while no input is provided by the shake tables. In this study, as presented later in the next sections, the actuators' forces are implemented using force control with force balance. When mass is present in the physical specimen as in the dynamic simulation, or partially in the quasi-dynamic simulation, the dynamic effect can be implemented either by the dynamic actuators, or by the shake tables and the actuators simultaneously, according to the criteria established by the users through the load splitting coefficient matrix. Therefore, the equation listed in the second line of Table 1 represents the unified formulation of Real Time Dynamic Hybrid Simulation, for which all seven cases shall produce the same response of the whole structure when subjected to ideal loading conditions. Note that the splitting coefficient matrices $\boldsymbol{\alpha}_m$ and $\boldsymbol{\alpha}_l(s)$ produce various testing methods of testing implying simultaneous numerical simulation in real time.

Test Platform for Real Time Hybrid Simulation

The basic test platform developed to implement the unified formulation for Real Time Dynamic Hybrid Simulation (see Fig. 2) is a force-based test platform (Shao, 2006). The platform uses multiple physical and computational systems including: (i) high-performance servo-hydraulic Structural and Seismic Testing Controllers; (ii) Data Acquisition and Information Streaming; (iii) Real Time Hybrid Simulation Controller that includes a computational model based Real Time Structure Simulator (RTSS); and (iv) force based unified System Compensation Controller (SCC). The idea of this test platform is to integrate and coordinate various hardware components during the hybrid simulation procedure and provide a realistic and efficient method to assess the performance of large-scale structural systems, or subassemblies or structural and non-structural components under real time earthquake loads. The platform developed herein allows also access of remote users to provide capabilities for geographically distributed testing over multiple laboratories. The platform is using a communication server (such as NTCP provided by the US George E. Brown Jr. Network for Earthquake Engineering Simulation-NEES), or can use a similar transfer protocol to stream the necessary data through the Internet to and from the Controller.

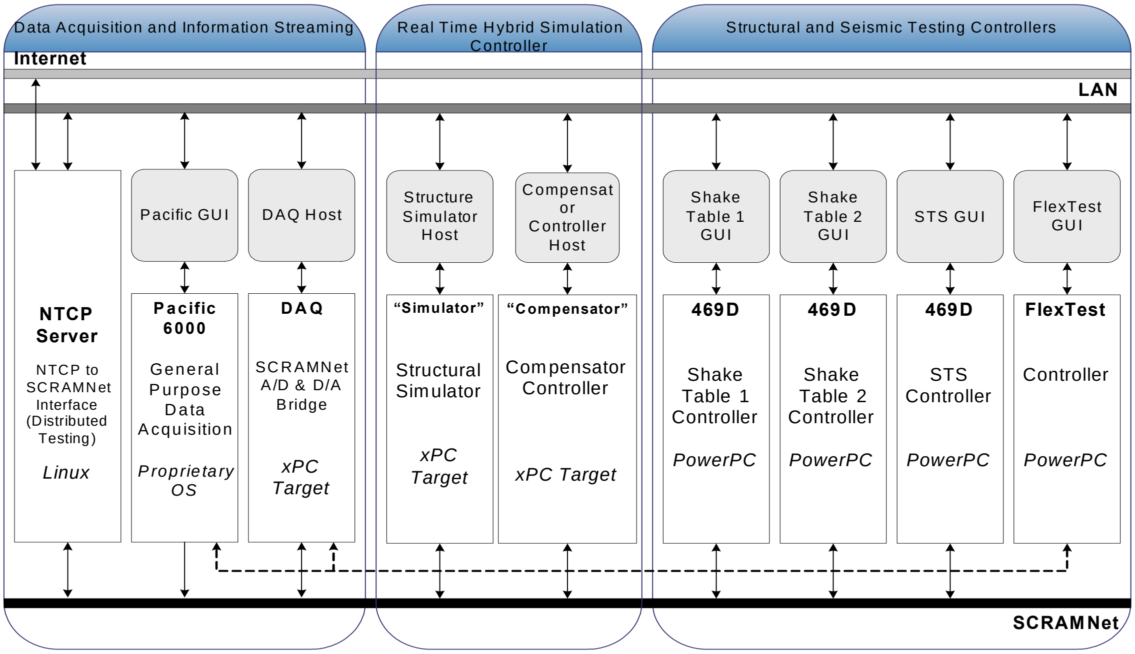


Figure 2. Hardware Components of Real Time Dynamic Hybrid Simulation test platform.

Among the three components in the platform, the Real Time Hybrid Simulation Controller (the middle box in Fig. 2) is of the most important in terms of defining the test methods, generating the proper drive commands for the loading devices (based on the numerical simulation result) and coordinating other components in the platform. A brief flow diagram of this controller including its sub-components and their functions in the platform is shown Fig. 3 and a detailed description is given in the following paragraphs.

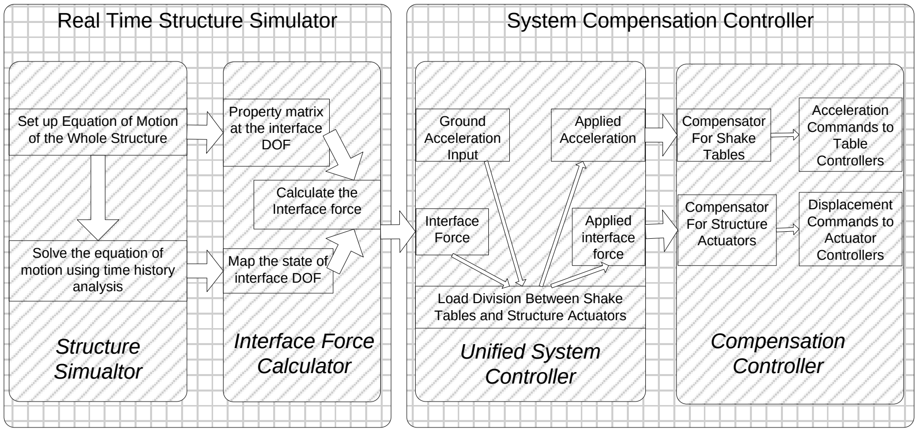


Figure 3. Flow diagram of Real time hybrid simulation controller.

Real Time Structure Simulator (RTSS)

The proposed Real Time Dynamic Hybrid Simulation (RTDHS) method requires a structure simulator to numerically solve the structural response of the *computational substructure* as well as to determine the interface forces applied to the *experimental substructure* in real time. It is referred to as the Real Time Structure Simulator (RTSS) in the test platform (see Fig. 3 right side box). The tasks performed by the RTSS include (i) the structural model assembly; (ii) the model condensation for real time simulation purpose; (iii) substructure partitions and numerical integration to obtain the simulated response. Interface forces are then calculated based on the simulated response of the computational substructure and the measured response from the experimental substructure. For more details about RTSS are presented in Shao (2006).

System Compensation Controller (SCC)

With the available interface forces from RTSS and the ground acceleration input pre-determined by the reseacher, the System Compensation Controller (SCC) performs two tasks as shown in Fig. 3. One is to regenerate the applied interface force and the applied acceleration command as described by the unified formulation. The Unified System Controller (USC) was developed to perform this function. Then the SCC needs to generate the final drive command for the dynamic acutators and the shake tables after a series of compensations, which were designed to eliminate the undesired effects due to actuator nonlinearities, time delay in response feadback, additional real, or virtual damping introduced in the test setup, etc.

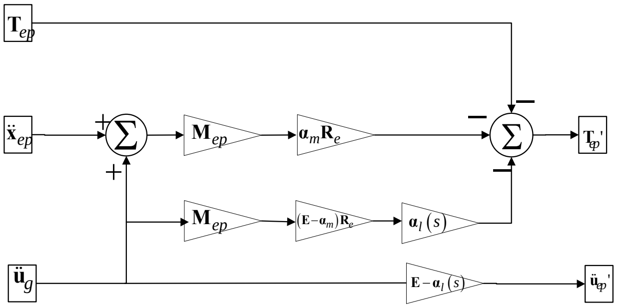


Figure 4. Unified System Controller flowchart.

Unified System Controller

The unified formulation (Eq. 6) can also be described in a flowchart as is shown in Fig. 4. The input to the USC must include: (1) the interface force $\mathbf{T}_{ep}$ calculated from the Real Time Structure Simulator; (2) the ground acceleration and (3) the acceleration response of the experimental substructure if a quasi-dynamic or a pseudo-dynamic simulation is defined by the user of the platform ($\boldsymbol{\alpha}_m \neq 0$). The load splitting coefficient matrix ($\boldsymbol{\alpha}_l(s)$) and the mass splitting coefficient matrix ($\boldsymbol{\alpha}_m$) of the experimental substructure are predefined and remain constant during the simulation. The output of the USC is the interface force and the acceleration that need to be applied by the dynamic actuators and shake tables respectively.

Compensation Controller

The successful implementation of the hybrid simulation technique requires the application of properly controlled dynamic actuators and shake tables so that they are able to apply the desired command to the specimen at the same loading rate. A compensation controller was developed for this purpose and the compensations necessary for both loading devices were identified and implemented.

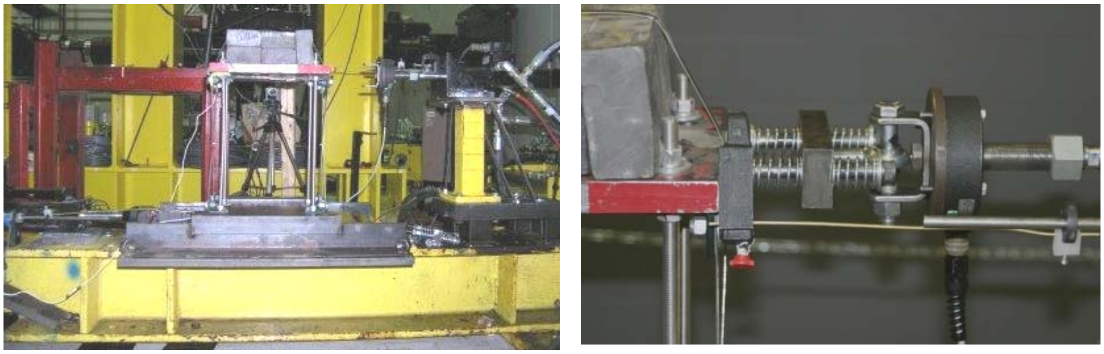
In other types of testing, such as traditional pseudo-dynamic testing, displacements (which are easier to control using servo-hydraulic actuators) are imposed on the specimen by actuators at rates lower than the dynamic response. In the simulation presented herein, control of displacements (deformations) of structures is more difficult since the experimental substructure is also undergoing base motion. Therefore, the first compensation necessary for the actuator is to allow the correct application of force magnitudes and rates. A method defined as “series elasticity actuators with displacement feedback” was proposed by Reinhorn *et al.* (2004) and adopted in the Compensation Controller. However it was found that this assembly introduces additional damping to the whole test setup, primarily due to velocity influence on the actuator’s response. To accommodate this additional damping, the Compensation Controller generates an additional damping force applied to the specimen besides the originally applied interface force. This additional damping force equals the product of the identified additional damping coefficients and the velocity response measured from the specimen. Finally the Compensation Controller compensates also the time delay effect introduced into the system by various components in the test platform, such as the hydraulic loading system, the data acquisition and data transferring system, and she structure simulator. A compensation procedure for time delay using “Smith’s Predictor” was development and verified (Shao, 2006).

A quasi-dynamic/pseudo-dynamic simulation requires an acceleration response feedback from the physical specimen in order to determine the next step acceleration input to the shake table. Inevitably, this feedback signal has time delay. The same time delay compensation using “Smith’s Predictor” can be used in the Compensation Controller for the shake table. In addition, when a hybrid simulation is defined by the user involving both shake table and actuators, each device exhibits different time delays in the implementation of the desired loading. This difference in the time delays must be identified and compensated so that both (or all) loading devices apply the desired loads at the same time instance. For example, assume that the response delay in the actuator (i.e. those used to apply interface forces in the hybrid testing) is smaller than of the shake table. Then the command send to the shake table should be delayed artificially by the value identified in the Compensation Controller for the shake table.

Verification Test

The concept of the proposed unified formulation for Real time Dynamic Hybrid Simulation and the corresponding test platform was experimentally verified using a small-scale pilot test setup as is shown in Fig. 5(A). The setup including a single degree of freedom frame structure, a dynamic actuator and a one unidirectional shake table. The mass of the structure is 174.4 lb (79.1kg) provided by a series of lead blocks. The resonant frequency is 3.03 Hz. By removing the lead bricks, a reduced mass specimen is obtained (only the mass of the small roof platform). This set-up is used for the demonstration of the quasi-dynamic hybrid simulation. The physical mass remaining in the reduced mass specimen is 23% of the total mass.

Fig. 5(B) shows the detail of the series connection between the actuator and the specimen. This series connection enables control of the actuator using force feedback while controlling piston position (displacement control) in the inner loop (Reinhorn *et al.* 2004). An MTS 252.22c two-stage servo-valve, 2.5gpm (9.5 l/min) and an MTS 204.09 actuator, 2.2kips (9.8kN) capacity, 4 inches (101.6mm) stroke, are used for the dynamic actuator. The actuator driving the shake table was fabricated by Parker Hannifin Corp and has a dynamic force rating of 21 metric ton and a dynamic stroke of 300 mm (±150 mm). Both actuator controllers were a generic MTS469-STS Real Time Hybrid Structural Test System (MTS, 2003) designed for NEES services of the Structural Engineering and Earthquake Simulation Laboratory (SEESL) at University at Buffalo. The System Compensation Controller was implemented in a generic closed loop system by using *Simulink* and *xPC Target* (Mathworks, 2006) and SCRAMNET^{TM}. Note that due to the limitation of the MTS controller, the actuator used to drive the shake table was also controlled in displacement instead of acceleration, which as more desired by the unified formulation.



(A) RTDHS TEST SETUP (B) SERIES SPRING CONNECTION DETAILS

Figure 5. Pilot Test Setup.

The unified formulation presented above allows several implementation cases to realize physically the dynamic loading in an experimental substructure. For the sake of test verification, the input to this single degree of freedom specimen consists only of a white noise base acceleration. This ground acceleration input was used in six different test configurations (see Table 2) using the splitting coefficient matrices described above in the unified formulation. In addition, a simple shake table test of the specimen was performed for comparison and reference.

Table 2. Parameters for test of the unified formulation for the Real Time Dynamic Hybrid Simulation.

TEST NAME	$\alpha(s)_I$	TABLE ACCELERATION INPUT	ACTUATOR'S APPLIED FORCES
Shake Table Test		$\ddot{u}_g$	None
Dynamic Test $\alpha_m = 0$	$\alpha(s)_I = 0$	0	$-M\ddot{u}_g$
	$\alpha(s)_I = 1.0$	$\ddot{u}_g$	P_2
	$\alpha(s)_I = 0.5$	$\alpha_I(s)\ddot{u}_g$	$-(1-\alpha_I(s))M\ddot{u}_g$
Quasi-Dynamic Test $\alpha_m = 0.77$	$\alpha(s)_I = 0$	0	$-M(\ddot{u}_g + \alpha_m\ddot{x})$
	$\alpha(s)_I = 1.0$	$\ddot{u}_{eq}$ *	0
	$\alpha(s)_I = 0.5$	$\alpha_I(s)\ddot{u}_{eq}$	$-(1-\alpha_I(s))M(\ddot{u}_g + \alpha_m\ddot{x})$

* $\ddot{U}_{eq} = (\frac{\ddot{u}_g + \alpha_m\ddot{x}}{1-\alpha_m})$ is the equivalent acceleration input of the shake table in a quasi dynamic test.

The white noise acceleration time series input was created by a function generator, using a frequency range of 0.1~10Hz and unity amplitude. The time series has a resolution of 1024 samples per second and a duration of 60 seconds. Since the object of this test is to verify the concept of the proposed unified formulation, it is desirable to protect the specimen from inelastic deformation during the simulation. Therefore the base motion amplitude scale is determined based on the simulation results obtained from a numerical model of the specimen to insure that the response would remain elastic and repeatable. As mentioned before, the actuator driving the shake table uses displacement control. The desired base acceleration was therefore doubly-integrated to obtain the displacement command before the test. An integration method was developed which first converts the acceleration signal from time domain to frequency domain using Fast Fourier Transformation. Subsequently, the frequency domain series is double integrated and an inverse Fourier Transform is applied to convert the data to a displacement time series. For low frequency accelerations, the above process can generate large displacement commands, which may exceed the actuator' stroke limit (~2 in). A high-pass cutoff frequency was set at 0.5 Hz prior to the double integration process in order to maintain the displacement command within system limits, which introduce error in test results that will be shown later in the test results.

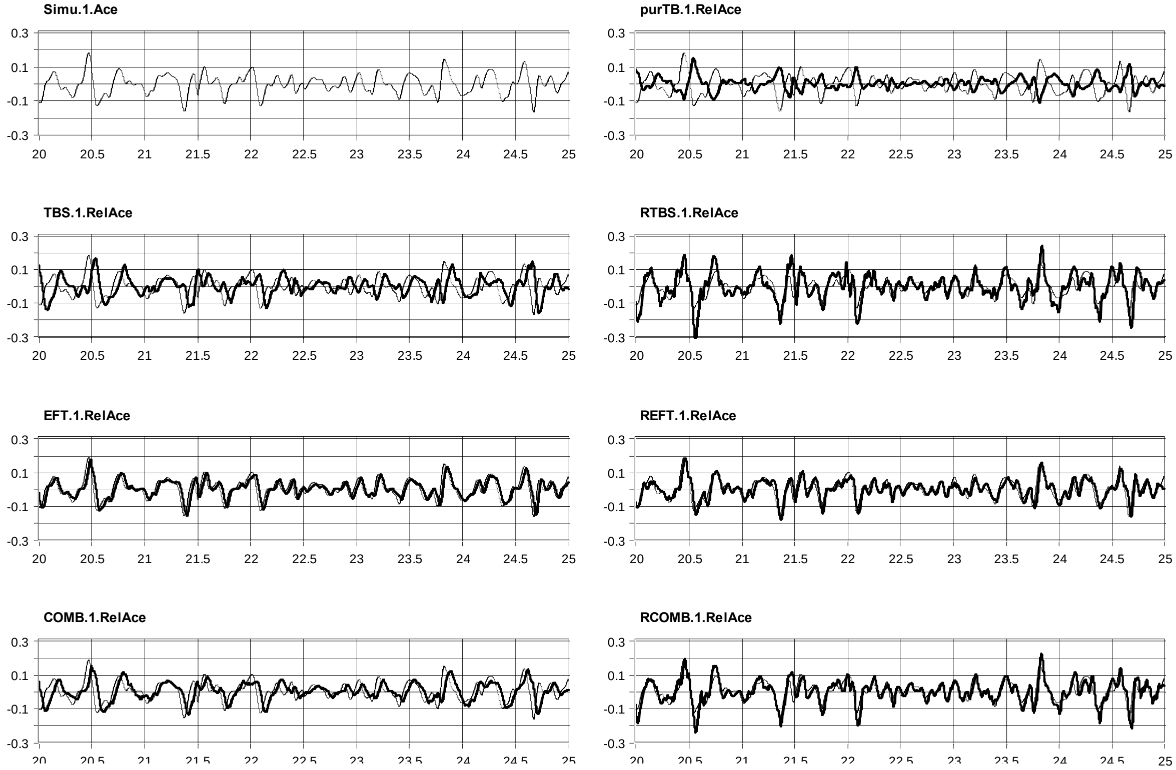


Figure 6. Measured structural displacement response compared to simulation results.

A series of sample test results is given in Fig. 6, where the measured structural displacement (thin line in the plot) from different implementation cases are compared with the simulation results (thicker line). The top left plot shows the structural displacement history obtained from a numerical simulation. The simulation response is computed using Simulink^{TM}, which calculates a continuous transfer function representing the dynamic properties of the specimen under the same white noise input. The top right plot is the measured data from the case in which the shake table is used while the dynamic actuator is not connected (first case listed in Table 2). The remaining three plots on the left are the response measured from tests with the full mass specimen representing a dynamic simulation, while the three right plots show the response measured from experiments with the reduced mass specimen. The latter cases are considered as pseudo-dynamic.

Except for the results obtained from the pure shake table test, all other measured displacement responses exhibit a good match to the simulated response, showing that different implementation cases derived from the unified formulation produce similar response in the specimen during the hybrid simulations. Therefore, the unified formulation is feasible and effective. The actuator is capable of applying the desired force to the specimen using the developed force control strategy. Compensations of additional damping and of time delay were also verified by the test results.

However, it is also notable that the response of the structure under shake table input only approximately matches the simulation results. This is due to poor tracking system of the makeshift shake table in displacement control rather than acceleration control. In the hybrid dynamic-pseudo-dynamic experiments, when a bigger mass is attributed to the virtual system, the displacement response consistently approaches the displacement computed in the dynamic simulation using the full mass specimen, with respect to the different implementation cases. The tests conducted using the actuator only show (i.e. EFT and REFT) best match to the numerical simulation (the third row in Fig. 6). In these cases, the results are nearly identical in amplitude of the numerical simulation response with a delay of approximately 13 milliseconds, which is equivalent to the time delay of the table actuator's response. Based on the these observations one may draw the conclusion that for a given specimen in a series of hybrid simulations, the errors increase when a larger portion of the dynamic load is generated by the shake table motion while less by the additional actuator.

Concluding remarks

A unified formulation is proposed for Real Time Dynamic Hybrid Simulation (RTDHS), which is a seismic simulation method which combines shake tables, dynamic actuators and numerical simulation in one test procedure. Using the two splitting coefficient matrices into the equation of motion of the experimental substructure, the unified formulation broadens the application range of RTDHS to include all the current modern seismic simulation methods. A corresponding test platform was developed to implement the unified formulation. Implementation issues were identified and possible solutions were proposed in this paper. Both the concept of the unified formulation and the test platform were verified experimentally by a simple one degree of freedom specimen hybrid simulation.

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